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UNSTEADY FLOW OF DUSTY VISCO- ELASTIC LIQUID BETWEEN TWO PARALLEL PLATES (KUVSHINISKI TYPE)

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ABSTRACT

The unsteady flow of dusty visco-elastic liquid (Kuvshiniski type) between two parallel plates when the lower plate is rest and the upper one begins oscillating harmonically in its plane is considered in view of its growing importance in various technical problems.

In the present paper, consider the unsteady laminar flow of visco-elastic (Kuvshiniski type) liquid containing uniformly small solid particles between two infinitely extended parallel plates when the lower plate is at rest and the upper one begins oscillating harmonically in its own plane. The analytical expressions for velocity fields of liquid and dust particles are obtained which are in elegant forms. The effects of elastic elements in the liquid, the mass concentration, and the relaxation time of dust particles on the velocity profiles are studied. The skin friction at the lower plate wall and the total volume flow in between the plates are also obtained.

KEYWORDS: Dusty fluid, laminar flow, Kuvshinisky type, visco-elastic liquid, elastic element, harmonic oscillation, skin friction.

1. INTRODUCTION

Interest in problems of flows of a dusty gas that is a mixed system of fluid and particles have increased enormously in recent years. A model equation describing the motion of such a mixed system has been given by P.G.Saffman. Based on Saffman's model, numerous authors investigated several dusty gas flow problems in different situations. In recent years, the study of non-Newtonian fluids has received special attention under a wide range of geometrical, dynamical, and rheological conditions. A few examples are the flow of nuclear fuel slurries, the flow of liquid metals and alloys such as the flow of gallium at ordinary temperatures (30 °c), the flow of plasma, the flow of mercury amalgams, handling of biological fluids, the flow of blood, a Bingham fluid with some thixotropic behaviours, coating of paper, petroleum production, plastic extrusion, molten paper pulp, emulsion, paints, lubrication with heavy oils and greases, aqueous solutions of polyacrylamide and polyisobutylene, etc, as important raw materials and chemical products in a large variety of industrial processes. The subject of Rheology is of great technological importance as in many branches of industry, the problem arises of designing apparatus to transport or to process substances which can not be governed by the classical stress-strain velocity relations. Visco-Elastic fluids are particular cases of non-Newtonian fluids that exhibit appreciable elastic behavior and stress-strain velocity relations and are time-dependent. Many common liquids such as oils, certain paints, polymer solutions, some organic liquids, and many new materials of industrial importance exhibit both viscous and elastic properties. Though the above fluids, called visco-elastic fluids, are also being studied extensively.

Saffman has expressed model equations describing the influence of dust particles on the motion of fluids. Several authors using equations of Saffman have investigated several dusty gas flow problems in different situations. Kapur and Sukla investigated the problem of two immiscible viscous liquids between two fixed parallel plates under a certain pressure gradient. The flow of visco-elastic Maxwell liquid down an inclined plane was investigated by Bagchi. The unsteady flow of two immiscible visco-elastic conducting liquids between two inclined parallel plates has been studied by Lahiri and Ganguly. Mandal et al have considered unsteady flow of dusty visco-elastic(kuvshiniski type) liquid between two oscillating plates. Johari et al have studied the MHD flow of a dusty Visco-elastic (kuvshiniski type) liquid past in an inclined plane. Recently Singh et al have studied the MHD flow of a dusty visco-elastic liquid past on an inclined plane.

Mathematical Formulation of the Problem and its Solution

We suppose that the dusty visco-elastic liquid fills the region between two horizontal infinite parallel flat plates at a distance h apart. The lower plate is kept at rest and the upper one begins to perform harmonic oscillations with a frequency ω in its own plane. The physical model is shown in figure-1. The present analysis takes a co-ordinate system such that the x-axis coincides with the lower fixed plate and the z-axis is perpendicular to it.

The dust particles are assumed to be spherical in shape and uniform in size and the number density of dust particles is taken as constant throughout the flow and it be ρ_0. Since the plates are infinite, the velocity will depend on z and time t mainly. For the constitutive equation we adopt Kuvshiniski type liquid, given by

$$P_{ij} = -p\delta_{ij} + p'_{ij}$$
$$\left(1+\lambda_0 \frac{D}{Dt}\right) p'_{ij} = 2\mu e_{ij}$$
$$\frac{D}{Dt} p'_{ij} = \frac{\partial p'_{ij}}{\partial t} + u_m \frac{\partial p'_{ij}}{\partial x_m} \quad \text{…..……………………..(1)}$$
$$2e_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$$

Where P_{ij} is stress tensor and p'_{ij} the deviatoric stress tensor, $\frac{D}{Dt}$ is the convective time derivative following a liquid element and u_i is the velocity of the liquid particle. Here λ_0 and μ denote the elastic coefficient and viscosity of the liquid. P.G.Saffman and using equation(1) we get the equation of motion of dusty visco-elastic liquid (dropping dashes)

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = R\frac{\partial^2 u}{\partial z^2} + \frac{f}{\tau}\left(1+\alpha\frac{\partial}{\partial t}\right)(v-u) \quad \text{……………………..(2)}$$
$$\frac{\partial v}{\partial t} = \frac{1}{\tau}(u-v) \quad \text{... …………………………(3)}$$

Here, $u' = \frac{u}{h\omega}$, $v' = \frac{v}{h\omega}$, $t' = \omega t$, $z' = \frac{z}{h}$, $\alpha = \lambda_0\omega$, $R = \frac{\gamma}{h^2\omega}$

Where, v is the dusty velocity of the particle.

f = mass concentration = $\frac{mB_0}{\rho}$

τ = relaxation time = $\frac{m\,\omega}{K}$

The relevant initial and boundary conditions in non-dimensional form are

$$t \le 0$$
$$u = \frac{\partial u}{\partial t} = 0 \text{ for all } z \quad \text{………………………………..(4)}$$
$$t > 0$$
$$u = a\,Sint \text{ at } z = 1$$
$$u = 0 \text{ at } z = 0 \quad \text{…………………………………(5)}$$

Taking $u = \underline{u(z)}\, e^{-bt}$, $v = \underline{v(z)}\, e^{-bt}$ $(b > 0)$ ………………………… (6)

Equations (2) and (3) takes the form

$$R\frac{d^2\underline{u}}{dz^2} + b(1-b\alpha)\underline{u} + \frac{f}{\tau}(1-b\alpha)(\underline{v}-\underline{u}) = 0 \quad \text{……………..…………………..(7)}$$

And $\underline{v} = \frac{1}{1-b\tau}\underline{u}$ ……………………………………(8)

Boundary conditions are $(t > 0)$

$$\underline{u} = a\,e^{bt}\,\text{Sint} \quad \text{at} \quad z = 1 \quad \text{…...……………………………...(9)}$$
$$\underline{u} = 0 \quad \text{at} \quad z = 0$$

Substituting equation (8) into equation (7) we get

$$\frac{d^2\underline{u}}{dz^2} - M^2\underline{u} = 0 \quad \text{……………………………………(10)}$$

Where, $M^2 = \frac{(b\alpha-1)bF}{R(1-b\tau)}$ and $F = 1 - b\tau + bf$

Thus the solution of the equation (10) is

$$\underline{u} = ACoshMz + BSinhMz \quad \text{……..……………………………(11)}$$

Using boundary conditions (9) we get

$$\underline{u} = \frac{a\,e^{bt}\,Sint}{SinhM} SinhMz \quad \text{for all } z \quad \text{………...……………………(12)}$$

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$$\underline{v} = \frac{a\, e^{bt}\, Sint}{SinhM} SinhMz \qquad \text{......................................(13)}$$

Therefore velocity profile of dusty fluids

$$u = \frac{aSint\ SinhM}{SinhM} \qquad (t > 0) \qquad \text{...................................(14)}$$

Velocity profile of dust particles

$$v = \frac{a\ Sint}{(1-b\tau)}\frac{Sinh}{SinhM}, \quad t > 0$$

The dimensionless shearing stress τ_p at the lower plate due to the dusty visco-elastic liquid is

$$\tau_p = \left[\left(1 - \alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial z}\right]_{z=0}$$

$$\tau_p = \frac{a\ M}{SinhM}\left(Sint - \alpha\, Cost \right)$$

$$\tau_p = \frac{a\left(\sqrt{k^2+1}\right)M}{SinhM} Sin(t - \theta_k), \quad \text{where } tan\theta_k = k \quad (k = 1,2,3,.....)$$

The volume flow of dusty Visco-elastic liquid discharged per unit breadth of the plate is given by

$$\phi = 2\int_0^1 u\, dz = \frac{2\, a\, Sint}{M\ SinhM}\left(CoshM - 1 \right)$$

For very small harmonic oscillations, velocity profile of the dusty visco-elastic fluid maintain the inequalities

$$\sum_{k=1}^{\infty} \frac{(M\, z)^{2k+1}}{(2k+1)\,!} \geq 0 \quad \text{or} \quad \sum_{k=1}^{\infty} \frac{(M\, z)^{2k+1}}{(2k+1)\,!} \leq 0$$

For very large harmonic oscillations, velocity profile of the dusty fluid maintain the inequalities

$$\sum_{k=1}^{\infty} \left(\frac{(M\,)^{2k+1}}{(2k+1)\,!} + \frac{(M\, z)^{2k+1}}{(2k+1)\,!}\right) \geq 0 \quad \text{or} \quad \sum_{k=1}^{\infty} \left(\frac{(M\, z)^{2k+1}}{(2k+1)\,!} - \frac{(M\,)^{2k+1}}{(2k+1)\,!}\right) \leq 0$$

2. RESULTS AND DISCUSSION:

The present analysis reveals that the solution of the given problem contains four pertinents non-dimensional parameters viz α(elastic parameter of the liquid particles), τ(relaxation time of dust particles), f(mass concentration of dust particles) and t(t is represented by ωt, harmonic oscillation). Behaviours of these parameters yield a physical insight into the problem. Numerically computation is made to observe the effects of these parameters on the velocity profile. Also, obtain the expression of skin friction at the lower plates and volume flow in between the plates.

Figures 1 depicts the velocity profile of fluid and dusty particles against z for different values of parameters when $\tau$and f are fixed at $\alpha = 1$.

Figures 2 depicts the velocity profile of fluid and dusty particles against z for different values of parameters when $\tau$and f are fixed at $\alpha = 2$.

Figures 3 depicts the velocity profile of fluid and dusty particles against z for different values of parameters when $\tau$and f are fixed at $\alpha = 3$.

Figures 4 depicts the velocity profile of fluid and dusty particles against z for different values of parameters when $\tau$and f are fixed at $\alpha = 4$.

The conclusions of the study -

(i)The velocity of dusty fluid increases as t increases and after attaining a maximum value near the plate,it increases as z increases.

(ii) The velocity of dusty particles decreases rapidly as t increases and after attaining a minimum value near the plate,it decreases as z increases.

(iii) Lastly it is also observed that from the graphs the magnitude of the velocity profile of dusty liquid and the magnitude of the velocity of dusty particles are identical.

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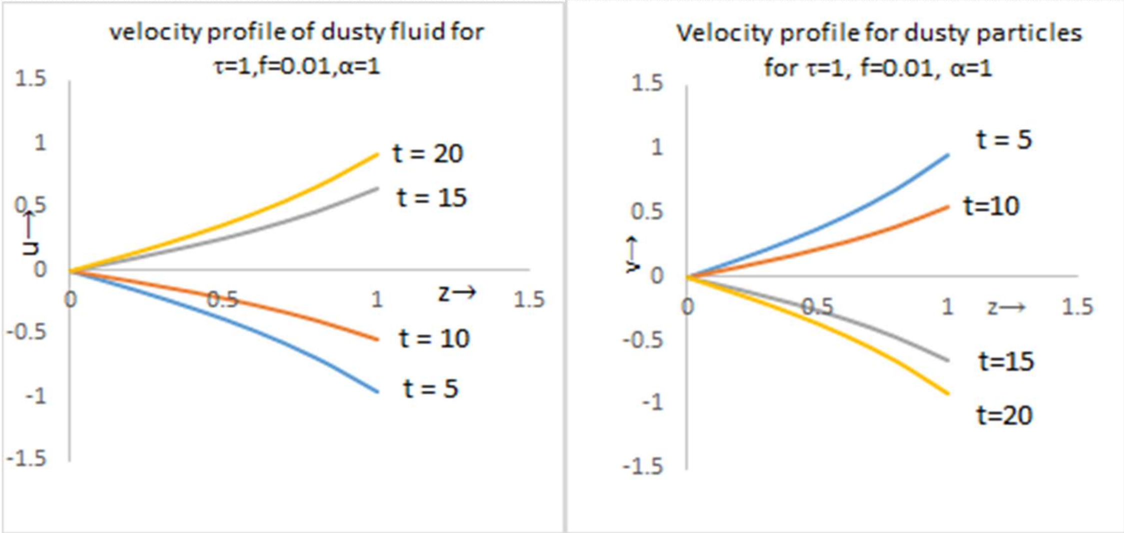

Figure - 1

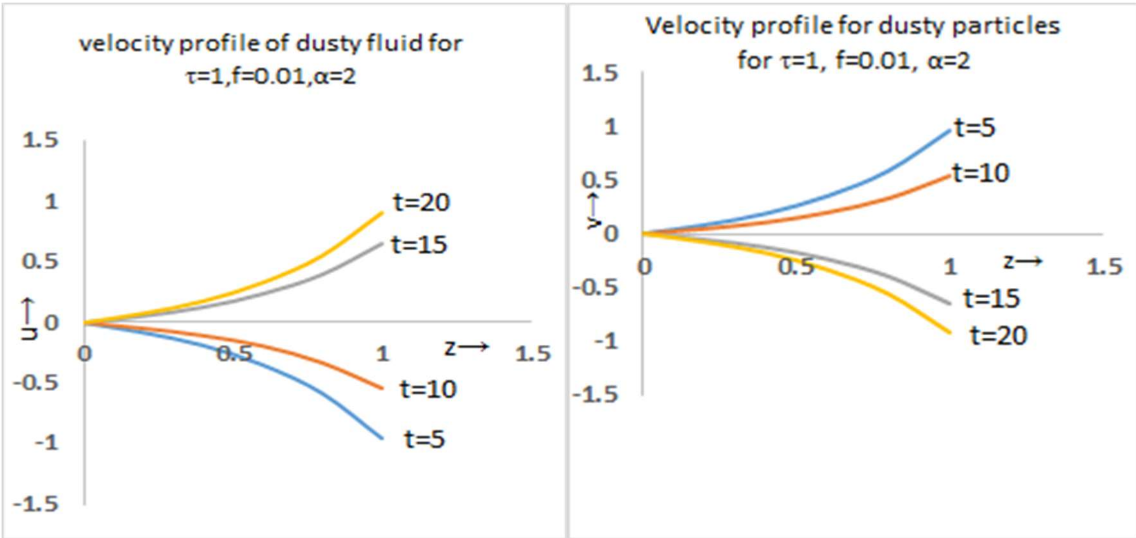

Figure - 2

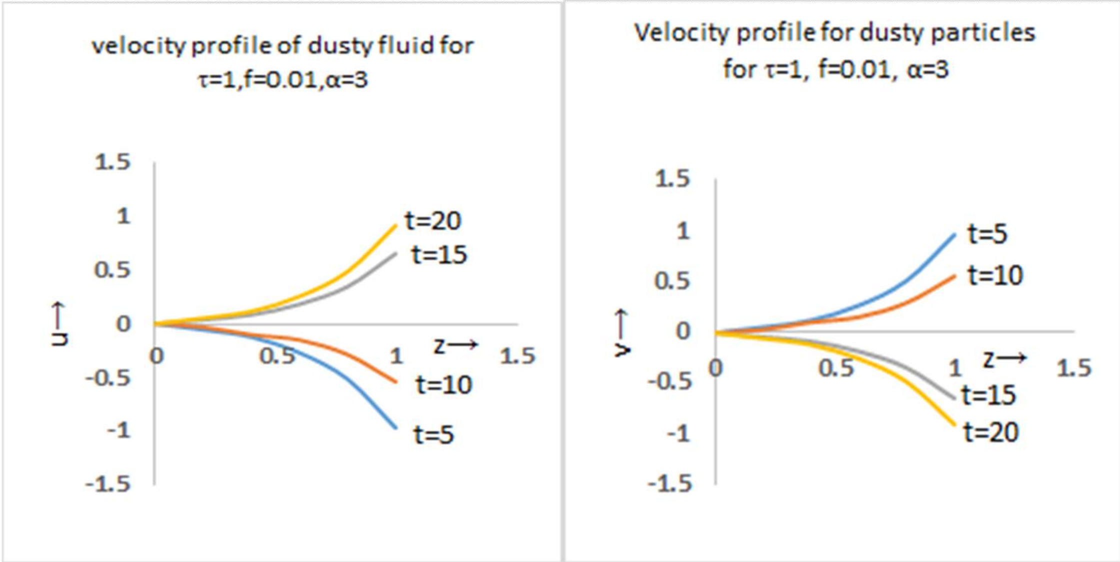

Figure-3

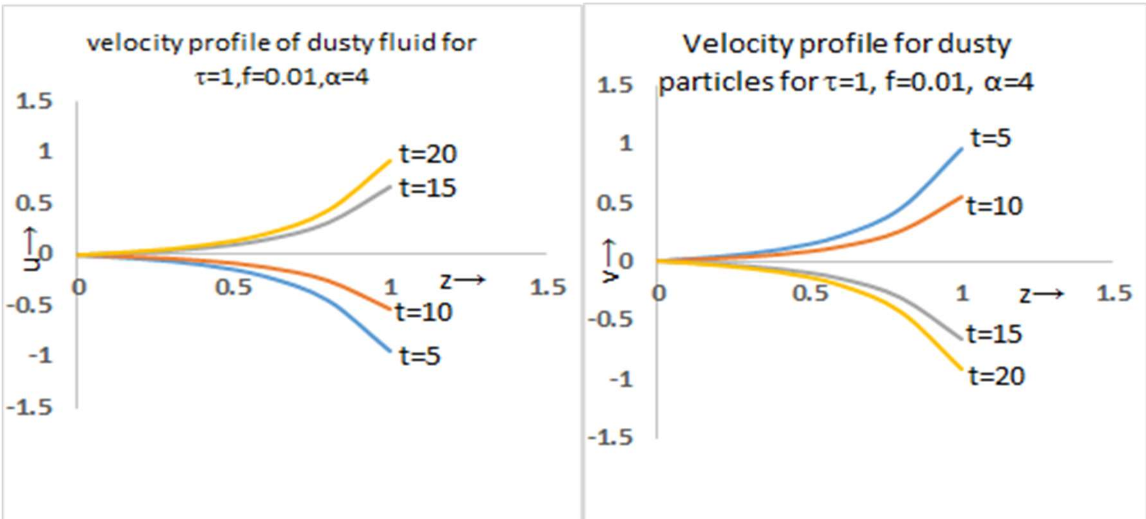

Figure-4

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REFERENCES

[1] Saffman P.G. - J.FluidMech. 13,120 (1962)
[2] Kapur K.C. - Ind.Jour.Mech,Math IIT , 2,108 (1965)
[3] Bagchi S. and Maiti M.K. - Acta Ciencia Indica, Vol.2,130 (1980)
[4] Mandal G.C.; Mukherjee S. and Mukherjee S - J.Indian Sci. 66,77 (1986)
[5] Johari, Rajesh and Gupta G.D. - Acta Ciencia Indica, vol XXV M No.3,275 (1999)
[6] Singh J.; Gupta C. and Varshney K.N.- Indian Journal of Theoretical Physics,vol-53,3,258(2005).